

# Investigating resonance in carbon-nanotube-reinforced composites

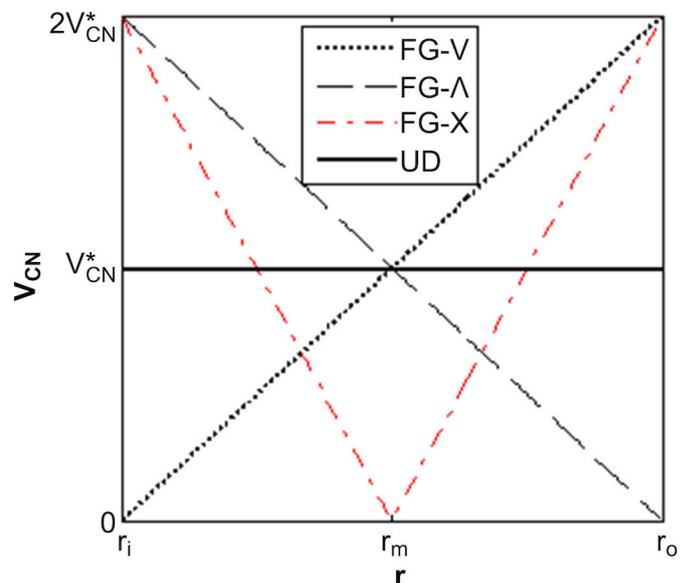
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*The waviness of single-walled carbon nanotubes has a significant effect on the vibrational characteristics of functionally graded axisymmetric cylinders.*

Since the invention of carbon nanotubes (CNTs)<sup>1</sup> more than 20 years ago, nanoscale engineering materials and technological applications have attracted a large amount of interest. For example, CNTs can be distributed according to specific grading patterns in a given direction to reinforce composite structures and thus improve their mechanical properties. Such reinforced composites are known as functionally graded CNT-reinforced composites (FG-CNTRCs). The high cost of CNTs, however, means that it is important to ensure the optimum amount of the reinforcements are used in polymer samples.

In a previous study, the material properties of CNTs were studied and the elastic modulus of composite structures reinforced with CNTs were determined through molecular dynamic simulations.<sup>2</sup> Furthermore, it has been shown that the CNT curvature (the waviness index) and aspect ratio have important influences on the mechanical properties and behavior of such composites.<sup>3</sup> In recent work, mesh-free methods have been used to investigate the vibrational and dynamic behavior of FG-CNTRC cylinders. For instance, we have previously conducted a dynamic analysis—using a mesh-free method—of FG nanocomposite cylinders that were reinforced with straight single-walled CNTs (SWCNTs).<sup>4</sup> We have also used the same mesh-free method to examine the effects of CNT orientation and aggregation on the vibrational and elastic wave propagation of FG-CNTRC cylinders.<sup>5,6</sup> Furthermore, in a closely related study, a radial base function mesh-free method was used to examine the elastic wave propagation of finite-length FG-CNTRC cylinders.<sup>7</sup> Most recently, we have studied the effects of CNT waviness and aspect ratio on the vibration and impact behavior of FG-CNTRC cylinders.<sup>8,9</sup>

In this work,<sup>10</sup> we continued our ongoing studies and used a mesh-free method to investigate the free vibration and resonance of



**Figure 1.** Illustration of the three linear functionally graded (FG) distributions and the uniform distribution (UD) of carbon nanotubes (CNTs), along the radial direction of the axisymmetric cylinder investigated in this study. The variation in the nanotube volume fraction ( $V_{CN}$ ) is shown as a function of the position along the cylinder radius ( $r$ ).  $r_i$ ,  $r_m$ ,  $r_o$ : Inner-radius, mid-radius, and outer-radius, respectively.  $V_{CN}^*$ : CNT mass fraction.

**Table 1.** First axisymmetric frequency parameter ( $\Omega_1$ ) of the homogeneous cylinders studied with the mesh-free method, in comparison with previously obtained results.<sup>11</sup>

| Grid arrangement | Method                    | $\Omega_1$ |
|------------------|---------------------------|------------|
| 5 × 15           | Mesh-free                 | 1.24700664 |
| 10 × 30          | Mesh-free                 | 1.24699705 |
| 20 × 60          | Mesh-free                 | 1.24699467 |
|                  | Zhou et al. <sup>11</sup> | 1.24699388 |

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**Table 2.** First three axisymmetric frequency parameters ( $\Omega_1$ ,  $\Omega_2$ , and  $\Omega_3$ ) of the nanocomposite cylinders.  $w$ : Waviness factor. The CNT distributions (UD, FG-V, FG- $\Lambda$ , and FG-X) are defined in Figure 1.

|            | $w = 0$ |        |               |        | $w = 0.425$ |        |               |        |
|------------|---------|--------|---------------|--------|-------------|--------|---------------|--------|
|            | UD      | FG-V   | FG- $\Lambda$ | FG-X   | UD          | FG-V   | FG- $\Lambda$ | FG-X   |
| $\Omega_1$ | 4.1178  | 4.0176 | 4.1051        | 4.1791 | 3.5124      | 3.5393 | 3.5459        | 3.5173 |
| $\Omega_2$ | 6.2009  | 5.9675 | 6.0374        | 6.3020 | 3.6940      | 3.7863 | 3.7844        | 3.7920 |
| $\Omega_3$ | 8.8683  | 8.5574 | 8.5406        | 8.9786 | 4.7668      | 4.8190 | 4.8430        | 4.7048 |

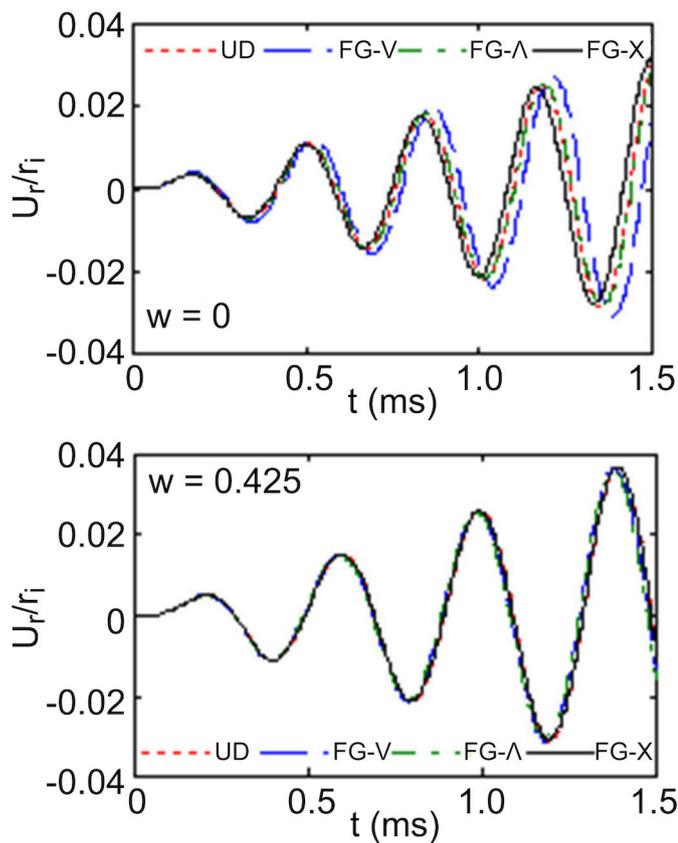
finite-length FG nanocomposite cylinders. We reinforced these cylinders with wavy SWCNTs and derived the axisymmetric frequencies of the cylinders. We subjected the cylinders to a periodic internal pressure, where the loading frequency was equal to the  $n^{\text{th}}$  natural frequency of the cylinder. For this work, we considered three different linear FG distributions, as well as a uniform distribution, of the wavy CNTs along the radial direction of the axisymmetric cylinders (see Figure 1).

To start, we used our mesh-free method to determine the first frequency parameter of a homogeneous solid cylinder that had a length/outer radius ( $L/r_o$ ) ratio of 4, a Poisson's ratio of 0.3, and free edges. The frequency parameters are a function of the natural frequency, outer radius, and the mechanical properties (i.e., the mass density and shear modulus) of the cylinder. Our results (see Table 1) are in good agreement with those obtained in a separate study<sup>11</sup> and thus indicate the high accuracy and convergency of our mesh-free method.

We have also considered CNTRC cylinders with clamped-clamped ends. These cylinders had a CNT volume fraction ( $V_{CN}$ ) of 0.17, an aspect ratio of 1000, an inner radius of 0.1m,  $r_o$  of 0.2m, and L of 0.4m. We investigated how the waviness ( $w$ ) and the distribution of the CNTs within the cylinder affected the frequency parameters (see Table 2). These results show that the FG-X-type cylinders, reinforced by straight (i.e.,  $w = 0$ ) CNTs, have the largest frequency parameter values in all modes. In contrast, the cylinders that were reinforced with wavy ( $w = 0.425$ ) CNTs exhibit irregular behavior. In addition, we find that the frequency parameters of the FG-CNTRC cylinders are greater than the UD cylinders.

We also show the first-mode radial vibration (resonance) of these cylinders (at 0.15m along the radius) in Figure 2. These results reveal that in cylinders containing straight CNTs, those with FG-V and FG-X distributions have the maximum amplitude and the maximum radial vibration speed, respectively. In cylinders with wavy CNTs, however, the FG-X distribution gives rise to the maximum amplitude and the FG-V distribution produces the maximum radial vibration speed.

In summary, we have used a mesh-free method to investigate the free vibration and resonance characteristics of axisymmetric cylinders containing different distributions of single-walled carbon nanotubes. We subjected these cylinders to a periodic internal pressure and examined the effect of three different functionally graded distributions, and a uniform distribution, of CNTs. Our results show that the degree of CNT waviness has a significant effect on the vibrational characteristics of these CNTRC cylinders. In the next stage of our work we wish to consider the effect of thermal loading on the thermostatic behavior of FG-CNTRC structures.



**Figure 2.** First-mode radial vibration ( $U_r$ )—measured at the mid-radius—of the FG CNT-reinforced composite cylinders that contain (top) straight (i.e.,  $w$  of 0) CNTs and (bottom) CNTs with  $w$  of 0.425.  $t$ : Time.

## Author Information

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